

# Data Integration, Simplified

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March 31, 2014

**Definition 1.** Let  $D$  be the set of all documents. Documents may belong to Source data or Target data.

**Definition 2.** Let  $T \subseteq D$  be a set of integration results, i.e. the Target set.  $T$  is generally considered to be the final result of compiling multiple Source sets through successive integration steps.

**Definition 3.** Let  $S \subseteq D$  be a set of Source documents considered for integration into a Target set.

**Definition 4.** Let  $rep_i : S, T_i \mapsto \{0, 1\}$  such that for  $x \in S, y \in T_i$ ,

$$rep_i(x, y) = \begin{cases} 1 & \text{if } x \text{ and } y \text{ represent the same document} \\ 0 & \text{otherwise.} \end{cases}$$

**Definition 5.** Let  $\Upsilon_i$  be a function that creates a document in  $T_{i+1}$  from a document of  $S$ . Formally, let  $\Upsilon_i : S \mapsto T_{i+1}$  such that for  $x \in S$ ,

$$\Upsilon_i(x) = y \mid y \in T_{i+1} \wedge rep_{i+1}(x, y) = 1$$

**Definition 6.** Let  $\Upsilon_i$  be a function that modifies a document from  $T_i$  using a document from  $S$  into  $T_{i+1}$ . Formally, let  $\Upsilon_i : S, T_i \mapsto T_{i+1}$  such that for  $x \in S, y \in T_i, rep_i(x, y) = 1$

$$\Upsilon_i(x, y) = z \mid z \in T_{i+1} \wedge rep_{i+1}(x, z) = 1$$

**Definition 7.** Let  $\overline{\tau}_i : S \mapsto T_{i+1}$  the integration function of  $x \in S$  with documents in  $T_i$  into  $T_{i+1}$ , such that  $\forall x \in S$ ,

$$\overline{\tau}_i(x) = \begin{cases} \Upsilon_i(x) = z & \text{if } \forall y \in T_i, rep_i(x, y) = 0. \\ \Upsilon_i(x, y) = y' & \text{if } \exists y \in T_i, rep_i(x, y) = 1. \end{cases}$$

**Lemma 1.** *The function  $\overline{\tau}_i$  is idempotent if and only if the subsequent integrations leads only to the  $\Upsilon$  functions. Formally,*

$$\overline{\tau}_i(x) \text{ idempotent} \iff \overline{\tau}_{i+1}(x) = \Upsilon_{i+1}(x, y)$$

*Proof.* If for  $x \in S$  and  $\forall y \in T_i, rep_i(x, y) = 0$ ,

$$\implies \overline{\tau}_i(x) = \Upsilon(x) = z \tag{1}$$

$$\implies \exists z \in T_{i+1} \mid rep_{i+1}(x, z) = 1 \tag{2}$$

$$\implies \overline{\tau}_{i+1}(x) = \Upsilon_{i+1}(x, z) \quad \square$$

If for  $x \in S, \exists y \in T_i \mid rep(x, y) = 1,$

$$\implies \bar{\tau}_i(x) = \bar{\mu}_i(x, y) = y' \quad (3)$$

$$\implies \exists y' \in T_{i+1} \mid rep_{i+1}(x, y') = 1 \quad \implies \bar{\tau}_{i+1}(x) = \bar{\mu}_{i+1}(x, y) \quad (4)$$