1. We will prove by induction on n
   INDUCTION HYPOTHESIS: assume that the claim is true when $n = k$, In other words, we assume that $\sum_{i=1}^{n} \lfloor \frac{i}{2} \rfloor = \lfloor \frac{k^2}{4} \rfloor$

   BASE CASE: k=1
   $\lfloor \frac{1}{2} \rfloor = \lfloor \frac{1^2}{4} \rfloor$
   $0 = 0$ thus the claim is true for the base case.

   INDUCTION STATEMENT: n=k+1
   $\lfloor \frac{k^2}{4} \rfloor + \lfloor \frac{k+1}{2} \rfloor = \lfloor \frac{(k+1)^2}{4} \rfloor$

   Now we can have two cases: k is either odd or k is even

   Case 1: k is even, thus k can be written as $2q$
   $\lfloor \frac{k^2}{4} \rfloor = \lfloor \frac{4q^2}{4} \rfloor = q^2$
   $\lfloor \frac{k+1}{2} \rfloor = \lfloor \frac{2q+1}{2} \rfloor = \lfloor q + \frac{1}{2} \rfloor = q$
   $\lfloor \frac{(k+1)^2}{4} \rfloor = \lfloor \frac{(2q+1)^2}{4} \rfloor = \lfloor \frac{(4q^2+4q+1}{4} \rfloor = q^2 + q$

   Thus by equating the two sides again we have
   $q^2 + q = q^2 + q$ and the claim is true when k is even.

   Case 2: k is odd, thus k can be written as $2q + 1$
   $\lfloor \frac{k^2}{4} \rfloor = \lfloor \frac{(2q+1)^2}{4} \rfloor = \lfloor \frac{4q^2+4q+1}{4} \rfloor = q^2 + q$
   $\lfloor \frac{k+1}{2} \rfloor = \lfloor \frac{2q+2}{2} \rfloor = q + 1$
   $\lfloor \frac{(k+1)^2}{4} \rfloor = \lfloor \frac{(2q+2)^2}{4} \rfloor = \lfloor \frac{(4q^2+8q+4}{4} \rfloor = q^2 + 2q + 1$

   Thus by equating the two sides again we have
   $q^2 + q + q + 1 = q^2 + 2q + 1$ and the claim is true when k is odd.

   Thus the claim is always true.

2. Proof by induction on n
   BC: n=1
   we can produce 1 by using the second fibonnaci number of 1. IH: assume the claim is true for all natural numbers $\leq k$
   IS: Now we want to prove that the claim is true when n=k+1.
   Consider the largest fibonnaci number less thank k+1 called f_1.
   We know that $f_1 + f_{i+1} = f_2 > k + 1$ which implies that $k + 1 - f_1 < f_{i-1}$
   Now since, we already assumed in the induction hypothesis that we can produce any natural number less than k using the sum of distint, non consecutive fibonnaci numbers, this implies that $k + 1 - f_{i-1}$ falls under the induction hypothesis and satisfies the constraint that they cannot be consecutive because f_{i-1} is not used. THus we have proven the claim

3. we will prove by induction on n
   Base Case: n=2 we have two questions: 1) is there a direct road from $A \rightarrow B$? 2) is there a

direct road from $B \to A$? thus we have $\leq 3(n-1) = 3$ questions
IH: assume that the claim is true for n cities, we want to prove that it is true for n+1 cities.
IS: Now when we have n+1 cities, we remove one and now we have n cities. It is assumed that we can solve the problem with (3n-1) questions, so when we add a city, we now get 3(n+1-1) = 3n questions questions. But we have already assumed that we know whether there is a deadend city or not so we must split it up into cases.

Case 1: Dead end city D existed for n cities, meaing there are (n-1) cities besies D and our newly added city (n+1) called X. So besides D, there are n cities now.
Questions: 1) is there a road going from D to A? if yes, D is not a Deadend. if no, D is still Deadend. 2) if yes, then we must ask if there is a road going from A to D. if yes, there is no deadend, if no, A could be dead end, but we must test this for (n-1) cities and therefore we must ask a total of (n-1) questions that ask is there a road from A to C for all cities C. Total amount of questions asked in Case 1 is (n+1).

Case 2: There is no deadend
A could be deadend, but we have to ask n questions: Is there a road from A to C for all cities C, but we already ask this in Case 1. So we can determine if there is a new city in under 3(n-1) questions thus proving the claim.

4. $\frac{15}{56}$
The sample space consists of all the series ending in 4 games plus all the games ending in 5, 6, and 7. This can be represente as $\binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4} = 56$ but we must multiply by 2 to include those scenarios in which either teams win. Now to find the probability of a team winning in 6 games we need to do $\frac{2*\binom{6}{4}}{112}$ which equals $\frac{15}{56}$

5. the probability that you can go from A to C is determined by the probability of going from A to C directly plus the probability of going from A to C through B.
p[A to C] = $(1-p)$
p[A to B to C] = $(1-p)(1-p)$ so we just add the two together to get
p[A to C] = $(1-p) + (1-p)^2$

6. The sample space = 100,000 numbers. Now to find out how many numbers would yield us a number divisible by 4, 6, or 9, we must use the principle of inclusion and exclusion.
$\frac{100{,}000}{4} + \frac{100{,}000}{6} + \frac{100{,}000}{9} - \frac{100{,}000}{24} - \frac{100{,}000}{54} - \frac{100{,}000}{36} + \frac{100{,}000}{216} = \frac{44445}{100000}$