Resolução 3.9

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1. Prove os seguintes sequentes usando dedução natural:

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\overline{\neg \exists x. P(x) \wedge Q(x)}$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\frac{\bot}{\neg \exists x.P(x) \wedge Q(x)}\ (Contr)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{Q(b) \qquad \neg Q(b)}{\bot}\ (And\text{-}I)}{\neg \exists x.P(x) \wedge Q(x)}\ (Contr)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{\dfrac{I}{Q(b)}\ () \qquad \dfrac{II}{\neg Q(b)}\ ()}{\bot}\ (And{-}I)}{\neg \exists x.P(x) \wedge Q(x)}\ (Contr)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\frac{}{Q(b)}\ ()$$

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$$\frac{P(b) \wedge Q(b)}{Q(b)}\ (And{-}E)$$

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$$\dfrac{\dfrac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)}\ (Exist-E)}{Q(b)}\ (And-E)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\frac{}{\neg Q(b)}\ ()$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\frac{P(b) \rightarrow \neg Q(b) \qquad P(b)}{\neg Q(b)}\ (Imp-E)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)}\ (All-E) \qquad P(b)}{\neg Q(b)}\ (Imp-E)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)}\ (All-E) \quad \dfrac{P(b) \wedge Q(b)}{P(b)}\ (And-E)}{\neg Q(b)}\ (Imp-E)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)}\ (All-E) \quad \dfrac{\dfrac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)}\ (Exist-E)}{P(b)}\ (And-E)}{\neg Q(b)}\ (Imp-E)$$

b) $\{ \forall P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x.\ P(x) \wedge Q(x)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)}\,(Exist-E)}{Q(b)}\,(And-E) \quad \dfrac{\dfrac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)}\,(All-E) \quad \dfrac{\dfrac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)}\,(Exist}{P(b)}\,(And-E)}{\neg Q(b)}\,(Imp-E)}{\bot}\,(And-I)}{\neg \exists x.P(x) \wedge Q(x)}\,(Contr)$$