Section 3.4: Problem 15

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January 22, 2014

Problem 15 Solution

Problem 15. The advantage of a certain blood test is that 90 percent of the time it is positive for patients having a certain disease. Its disadvantage is that 25 percent of the time it is also positive in healthy people. In a certain location 30 percent of the people have the disease, and anybody with a positive blood test is given a drug that cures the disease. If 20 percent of the time the drug produces a characteristic rash, what is the probability that a person from this location who has the rash had the disease in the first place?

What we know:

For patients with the disease: 90 percent test positive, leaving 10 percent to test negative
For patients without the disease: 25 percent test positive, leaving 75 percent to test negative
In the location, 30 percent have the disease
All who test positive are given the treatment, which causes a rash 20 percent of the time

What we need to figure out:

What is the probability that a person (from this location) who has the rash also initially had the disease?

Let's begin:

We can ignore the information that 20 percent of the time the drug produces a characteristic rash, as we assume that everyone given the drug (whether they have the disease or not) has an equal chance of developing a rash.

Now, lets determine the proportion of people who receive the drug AND have the disease:

30 percent have the disease, 90 percent of who will test positive.

$$.30 \times .90 = .27$$

So 27 percent of the population has the disease and will test positive

Next, lets determine the proportion of people who receive the drug BUT DO NOT have the disease:

70 percent do not have the disease, but 25 percent will test positive anyway.

$$.70 \times .25 = .175$$

So 17.5 percent of the population has the disease and will test positive

Now, we'll find the total proportion of the population that will receive the drug by adding the proportion of the population that has the disease and tests positive to the proportion of the population that does not have the disease but has a false positive test

$$.25 + .175 = .445$$

So 44.5 percent of the population will test positive and receive the drug.

Finally, to find the probability that a person who has the rash (or in this case received the drug) also had the disease in the first place, we divide the proportion of the population that tested positive for the disease and had the disease by the proportion of the population that tested positive for the disease:

$$\frac{.27}{.445} = .61 \tag{1}$$

This shows that a person from this location who has a rash has a probability of .61 of also having the disease in the first place.