Simple Mathematical Induction

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1 Introduction

With this paper we will establish formula by the use of mathematical induction. An it will be a step by step way to solve. An hopefully it will help out the reader

2 Formula

We are given the formula

$$\sum_{n=1}^{\infty} i = \frac{n(n+1)}{2}, \forall n \geq 1$$

And n is element of the positive integers.

3 Proof

We began by running off a few tern to aid us in seeing a pattern emerge in the formula.

$$\sum_{n=1}^{\infty} 1 + 2 + 3 + 4 + 5 + \cdot + n = \frac{n(n+1)}{2}$$

4 Initial Step

Let's assume that n=1,

$$1 = \frac{1(1+1)}{2} = 1$$

$$1 = \frac{2}{2} = 1$$

Next we assume that n=k and k is an element of the positive integers. And we denote it as equation 1.

$$\sum_{k=1}^{\infty} 1 + 2 + 3 + 4 + 5 + \cdots + k = \frac{k(k+1)}{2} \tag{1}$$

$$\sum_{k=1}^{\infty} 1+2+3+4+5+\cdots+k = \frac{k(k+1)}{2}$$

Now we do k+1 step and use equation 1 to help us solve the proof.

$$\sum_{k=1}^{\infty} 1+2+3+4+5+\cdots+k+k+1 = \frac{(k+1)((k+1)+1)}{2}$$

To aid us in seeing the proof we will simplify the right hand side (RHS) of the equation.

$$\sum_{k=1}^{\infty} 1+2+3+4+5+\cdots+k+k+1 = \frac{(k+1)(k+2)}{2}$$

From (1) we will use in the Induction hypothesis t o prove the formula.

$$\frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$$

The next step we do is find the greatest common factor (GCF) on the left hand side (LHS).

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

Now we can combine the (LHS) of the equation since we have the same denominator.

$$\frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

Next we factor the equation on the (LHS).

$$\frac{k^2+k+2k+2}{2} = \frac{(k+1)(k+2)}{2}$$

Combine like terms on the (LHS).

$$\frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$$

Final we factor the equation on the (LHS) and we will be done.

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

Thus we achieved what we desired¿