Complex Numbers

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1 Warm-Up Set

1. Let $z^4 = 1$. (a) What are the four complex roots of z? (b) Draw the roots of z on the complex plane.

2. Now let $z^5 = 1$. (a) What are the five complex roots of z? (b) Draw the roots of z on the complex plane.

3. Let w be a complex number such that $|w| = 3$. Find the largest possible value of $|i + 1 - w|$.

2 Practice

1. The complex number z is equal to $9 + bi$, where b is a positive real number and $i^2 = -1$. Given that the imaginary parts of z^2 and z^3 are the same, what is b equal to?

2. There is a complex number z with imaginary part 164 and a positive integer n such that $\frac{z}{z+n} = 4i$. Find n.

3. Let $P(z) = x^3 + ax^2 + bx + c$, where a, b, and c are real. There exists a complex number w such that the three roots of $P(z)$ are $w + 3i$, $w + 9i$, and $2w - 4$, where $i^2 = -1$. Find $|a + b + c|$.

4. Let $z = a + bi$ be the complex number with $|z| = 5$ and $b > 0$ such that the distance between $(1 + 2i)z^3$ and z^5 is maximized, and let $z^4 = c + di$. Find $c + d$.

5. The complex numbers z and w satisfy $z^{13} = w$, $w^{11} = z$, and the imaginary part of z is $\sin \frac{m\pi}{n}$, for relatively prime positive integers m and n with $m < n$. Find n.

6. (Challenge Problem) Complex numbers a, b, and c are zeros of a polynomial $P(z) = z^3 + qz + r$, and $|a|^2 + |b|^2 + |c|^2 = 250$. The points corresponding to a, b, and c in the complex plane are the vertices of a right triangle with hypotenuse h. Find h^2.