Assignment 1

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Question 1

Research has shown that in a certain language, the distribution of the number of letters in words in texts is close to Poisson with parameter 4.

Use the central limit theorem to approximate the probability that a text of 1000 words has more than 4100 letters. Explain explicitly what assumptions you are making to guarantee that this approximation can be used.

Answer: We follow the four steps:

1. We need to assume that subsequent words are i.i.d., and that their expectation and variance are bounded.
2. The lengths of words are Poisson with parameter 4, so that the mean and the variance are 4.
3. We are asked to compute the probability that a text of 1000 words has more than 4100 letters: $\mathbb{P}(S_{1000} > 4100)$, where S_{1000} is the number of letters in the text of 1000 words.

   We rewrite this as

   $$\mathbb{P}(S_{1000} > 4100) = \mathbb{P}\left(\frac{S_{1000} - 4100}{\sqrt{1000 \cdot 4}} > \frac{4100 - 4000}{\sqrt{1000 \cdot 4}}\right) \approx \mathbb{P}(Z_{1000} > 1.58112).$$

4. CLT approximation: We approximate

   $$\mathbb{P}(S_{1000} > 4100) \approx \mathbb{P}(Z_{1000} > 1.58112) \approx \mathbb{P}(Z > 1.58112) \approx 0.0571.$$

Question 2

An urn contains 2 black and 3 white balls. We repeatedly draw a ball, and replace it with two balls of the same color. We assume that all balls are drawn with equal probability.

2A

Compute the probability of drawing first a black and then two white balls in the first three draws. Also compute the probability of first drawing two white balls and then the black ball.

Answer: Write BWW for the sequence of first a black and then two white balls in the first three draws (and similarly with WWB etc). Then

$$\mathbb{P}(BWW) = \mathbb{P}(B)\mathbb{P}(BW|B)\mathbb{P}(BWW|BW) = \frac{2}{5}\frac{3}{6}\frac{4}{7} = \frac{4}{35}.$$

2B

Compute the probability of drawing a white ball in the first draw. Also compute the probability of drawing a white ball in the first draw conditionally on drawing a black ball in the second draw. Give an interpretation of why these are different.

Answer: We start by noting that $\mathbb{P}(W) = \frac{3}{5}$. Further, let B_2 denote the event that we draw a black ball in the second draw. Then we need to compute the conditional probability $\mathbb{P}(W|B_2)$. We do this by writing

$$\mathbb{P}(W|B_2) = \frac{\mathbb{P}(WB)}{\mathbb{P}(B_2)} = \frac{\mathbb{P}(WB)}{\mathbb{P}(WB) + \mathbb{P}(BB)}.$$

Then we compute

$$\mathbb{P}(W|B_2) = \frac{\frac{3}{5}\frac{2}{6}}{\frac{3}{5}\frac{2}{6} + \frac{2}{5}\frac{3}{6}} = \frac{6}{12} = \frac{1}{2}.$$

This probability is smaller than $\mathbb{P}(W) = \frac{3}{5}$, since drawing the black ball in the second draw makes it more likely that we had drawn a black ball in the first draw.