MATH 302-01 **Breanna Rodriguez**
Prof. Soto Due Date: 08/25/19

Homework 0

STATEMENT #1:

Prove that
$$1 + 2 + 3 + ... + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}. \tag{1}$$

Proof. We would like to prove that $\sum_{k=1}^{n} k = \frac{n(n+1}{2}$ so we proceed by induction.

Base Case. For $n = 1$ we see that the left hand side of (1) is 1 whereas the right hand side is given by
$$\frac{1(1+1)}{2} = 1.$$
as well. Hence the statement is true for $n = 1$

Induction Case. Assume that the statement holds for n+1, that is we need to show that
$$\begin{aligned} 1 + 2 + 3 + \ldots + n + (n+1) &= \frac{(n+1)((n+1)+1)}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$
Thus we will begin with the left hand side of (1) to reach our conclusion. By our assumption we know that
$$1 + 2 + 3 + \ldots + n + (n+1) = \frac{n(n+1)}{2} + (n+1)$$
Thus we use a bit of algebra as follows to reach our conclusion:
$$\begin{aligned} 1 + 2 + 3 + \ldots + n + (n+1) &= \frac{n(n+1)}{2} + (n+1) \\ &= \frac{n(n+1)}{2} + \frac{2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2}. \end{aligned}$$
Thus by induction we see that statement (1) is true.

$\square$